Using Intuitive Geometry - Exercise 7

Due Date: October 13th - Instructor: Felix Breuer

Exercises

1) The *Eight Queens Problem* is this: Place eight queens on a chess board such that no queen can capture any other in one move.

Construct a polytope such that the lattice points in this polytope are in bijection with the solutions to the Eight Queens Problem.

2) Prove that

$$L(k\Delta_i^d) = L((k-i)\Delta^d) = \binom{k+d-i}{d}$$

for any $i < k \in \mathbb{Z}$ and $i \leq d+1$ where

$$\Delta_i^d = \{x \in \mathbb{R}^{d+1} | \sum_i x_i = 1, x_1 > 0, \ldots, x_i > 0, x_{i+1} \geq 0, \ldots, x_{d+1} \geq 0\}.$$

You can use that $L(k\Delta^d) = \binom{k+d}{d}$ as we proved in class.

3) Let K be a shellable simplicial complex. Let v be any vertex of K. Show that $\mathrm{star}(v)$ and $\mathrm{link}(v)$ are shellable. Here

$$\mathrm{star}(v) = \{\sigma \in K | \exists \sigma^* \in K : v \in \sigma^* \text{ and } \sigma \subset \sigma^*\},$$

$$\mathrm{link}(v) = \{\sigma \in \mathrm{star}(v) | v \notin \sigma\}.$$

4) Let P denote the 3-dimensional octahedron. Compute the Ehrhart polynomial $L_P(k)$ of P. Proceed as follows:

1. Construct a unimodular triangulation T of P.
2. Construct a shelling of T.
3. For any k, count the number of simplices of type k in your shelling.
4. Apply the formula $L_P(k) = \sum_{i=0}^{d} h_i \binom{k+d-i}{d}$.

Note: We constructed a shelling of the 2-dimensional boundary complex of the octahedron in class. You are supposed to triangulate the full 3-dimensional octahedron.

5) Draw a picture of the shelling you constructed in 4).

Optional Problems

A) Give a solution to the Eight Queens Problem.

B) Prove that Bing's house with 2 rooms is not shellable.

Questions?

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