Using Intuitive Geometry - Exercise 3

Due Date: September 15th - Instructor: Felix Breuer

Exercises

1) Solve the following system of linear equation using Gaussian Elimination *and visualize each step* by drawing the three hyperplanes and their lines of intersection in each step. You can either draw these by hand or use a computer.

$$\begin{pmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 3 & 1 & 1 \end{pmatrix} x = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$$

2) Find a solution of the following system of linear inequalities using Fourier-Motzkin Elimination.

$$\begin{pmatrix} -1 & -4 \\ -2 & -1 \\ 1 & -2 \\ 1 & 0 \\ 2 & 1 \\ -2 & 6 \\ -6 & -1 \end{pmatrix} x \leq \begin{pmatrix} -9 \\ -4 \\ 0 \\ 4 \\ 11 \\ 17 \\ -6 \end{pmatrix}$$

The remaining exercises lead toward a proof of the fact that H-polyhedra are V-polyhedra.

3) Show that every H-polyhedron $P = \{x | Ax \leq b\}$$ can be written as the intersection of a V-polyhedron (in fact, a V-cone) with an affine subspace.

- Write P as the intersection of an H-cone with an affine subspace.
- Come up with a V-representation of this cone.

4) Show that *if* you know that the intersection of a V-polyhedron with a coordinate hyperplane is a V-polyhedron, *then* you know that the intersection of a V-polyhedron with an arbitrary affine subspace is a V-polyhedron.

The last task is to get some intuition about the last step in the proof, namely to show that the intersection of a V-polyhedron with a coordinate hyperplane is a V-polyhedron.

5) Given a V-poly*tope* $P = \mathbf{conv}(V) \subset \mathbb{R}^d$ and the coordinate hyperplane $H_d = \{x | x_d = 0\}$, come up with a finite set of points $W \subset H_d$ such that $\mathbf{conv}(W) = P \cap H_d$. Hint: Draw a picture and then let yourself be inspired by Fourier-Motzkin Elimination.

Note: You do *not* have to prove that $\mathbf{conv}(W) = P \cap H_d$ actually holds. Your job in problem **5)** is just to write down the correct set W.

Optional Problems

A) Prove $\mathbf{conv}(W) = P \cap H_d$.

B) In the lecture, we saw both an "intuitive" and a formal proof of LP Duality. Can you come up with an elegant way of making the "intuitive" proof formally precise? In particular, what is the best way to prove that if x is an optimal vertex, then c is a non-negative combination of the normals of those defining hyperplanes that x is contained in?

Questions?

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